A Novel Wide Band Absorbing Boundary Condition for FDTD Method Simulations

Ao Sheng Rong* Zhong Liang Sun+ Vijai K. Tripathi* Hua Xin Wang+ and Wen Bin Dou+

* *Department of Electrical and Computer Engineering*
Oregon State University, Corvallis, OR 97331
+ *State Key Laboratory of Millimeter Waves, Department of Radio Engineering*
Southeast University, Nanjing 210096, P. R. China

ABSTRACT

A wide band absorbing boundary condition that utilizes the basic relations between transverse fields of a guided travelling wave in the frequency domain is presented for FDTD method simulations. The resultant ABC operator ensures that the outgoing waves on the boundaries can be absorbed over the total frequency band of interest. The exponential approximation is used to recursively implement our ABC in the FDTD simulation, avoiding the need for the complete time history of field components. The numerical experiments are used to demonstrate that the present ABC consistently has a superior absorbing performance over a wide frequency band.

INTRODUCTION

The FDTD method is being used increasingly to characterize microwave and digital integrated circuits and packages. Because these circuits are typically open region problems, absorbing boundary conditions (ABCs) must be employed to terminate the computation space. Many types of absorbing boundary conditions have been proposed [1-4], among which the Mur's first order ABC seems to be widely used. However, all these ABCs were designed to assure that a wave is absorbed without reflection for specific cases only. Recently, a new ABC based on the use of a perfect matched layer was presented [5]. This ABC, however, is valid for free space simulations only.

In this paper, we propose a novel and quite different version of ABC for FDTD method simulations. Our idea stems from the use of the basic relationships between transverse fields of a guided travelling wave in the frequency domain. The resultant ABC operator ensures that the outgoing waves on the boundaries can have significant absorption over the total frequency band of interest. This implies that our ABC is essentially wide band. The exponential approximation is used to recursively implement our ABC in the FDTD simulation, avoiding the need for the complete time history of field components. The numerical experiments show that the present ABC has consistently superior absorbing performance over a wide frequency band.

WE 3D

FORMULATION

The formulation of our ABC is summarized as follows.

For a guided wave travelling along +z direction without reflection, the basic relationship between transverse electric fields can be expressed in the frequency domain as:

$$\overline{E}_t(z+\Delta z,\omega) = \Phi(\omega)\,\overline{E}_t(z,\omega) \qquad (1)$$

$$\Phi(\omega) = e^{-j\beta\Delta z} \tag{2}$$

where β is the propagation constant and may be thought as a known parameter. For uniform propagation structures, it involves solving a 2-D electromagnetic problem and can be computed in advance by using well-known spectral domain approach or 2-D FDTD method. From the physical point of view, eqn.(1) states that a wave will undergo a phase delay of $\beta\Delta z$ when it propagates from z to $z+\Delta z$.

When eqn.(1) is analytically extended from the frequency domain into the s domain $(s=j\omega)$, we have

$$\overline{E_t}(z+\Delta z,s) = \Phi(s)\ \overline{E_t}(z,s) \tag{3}$$

To ensure the stability of solutions, the poles of the phase factor $\Phi(s)$, if any, have to be located in the left half s-plane.

Making the inverse laplace transformation of eqn.(3) yields

$$\begin{aligned}\overline{E_t}(z+\Delta z,t) &= \int_0^t \Phi(t-\tau)\ \overline{E_t}(z,\tau)d\tau \\ &= \int_0^t \Phi(\tau)\ \overline{E_t}(z,t-\tau)\ d\tau\end{aligned} \tag{4}$$

Let $t = n\Delta t$ and assume all field components to be constant over each time interval Δt. So we have

$$\overline{E_t^n}(z+\Delta z) = \sum_{m=0}^{n-1} \Phi^m\ \overline{E_t^{n-m}}(z) \tag{5}$$

$$\Phi^m = \int_{m\Delta t}^{(m+1)\Delta t} \Phi(\tau)\ d\tau \tag{6}$$

Eqn.(5) indicates that the transverse fields at node $z+\Delta z$ at time $n\Delta t$ may be expressed in terms of those at node z for $t \le n\Delta t$. At a glance, the convolution summation in eqn.(5) requires complete time history of transverse fields at node z for $t \le n\Delta t$. As will be shown in latter, however, if a decaying series of exponential functions are used to approximate $\Phi(t)$ (i.e. so called Prony's approximation), eqn.(5) may be evaluated in a recursive manner.

We approximate $\Phi(t)$ in terms of a decaying series of exponential functions as

$$\Phi(t) = \sum_{k=1}^{K} c_k\ e^{-\alpha_k n\Delta t} \tag{7}$$

where the expansion coefficients c_k's and the exponential factors α_k's are usually complex. They can be obtained by following the standard approximation procedure [6]. Substituting eqn.(7) into (6) yields

$$\Phi^m = \sum_{k=1}^{K} \frac{c_k}{\alpha_k}(1-e^{\alpha_k\Delta t})e^{\alpha_k m\Delta t} \tag{8}$$

In order to derive the recursive ABC operator, we substitute eqn.(8) into (5) and make proper manipulations similar to [7], leading to the following recursive relations:

$$\overline{E_t^n}(z+\Delta z)=\sum_{k=1}^{K} b_k\overline{E_t^n}(z)+\sum_{k=1}^{K}\overline{\Psi_k^n}(z) \tag{9}$$

$$\overline{\Psi_k^n}(z)=b_k e^{-\alpha_k\Delta t}\overline{E_t^{(n-1)}}(z)+e^{-\alpha_k\Delta t}\overline{\Psi_k^{(n-1)}}(z) \tag{10}$$

where

$$\overline{\Psi_k^1}(z) = \overline{\Psi_k^0}(z) = 0$$

$$b_k = \frac{c_k}{\alpha_k}(1-e^{-\alpha_k\Delta t})$$

Eqn.(9) permits the transverse fields at node $z+\Delta z$ at time $n\Delta t$ to be recursively updated. It should be mentioned that because $\Phi(t)$ is real, if one exponential factor α_k in eqn.(7) is complex, then its conjugate α_k^* must be involved. The expansion coefficients c_k and c_k^* corresponding to α_k and α_k^* also appear in conjugate form. This property of c_k's and α_k's allows us to reduce the summation terms in eqn.(9) with respect to the index k by almost the half.

NUMERICAL EXPERIMENTS

Our new ABC has been tested by implementing it for the FDTD simulation of a Ka-band rectangular waveguide. A synthetic excitation containing a great number of sinusoidal waves over the frequency band of interest has been used. The ideal reference signal without reflection is obtained by letting the end plane far away from the source plane. For a rectangular waveguide, it can be easily shown that

$$\Phi(t) = \delta(t-t_0) - \phi'(t-t_0) \qquad (11)$$

where $t_0 = \Delta z/c$ and c is the light velocity in the free space. Fig.1 shows the accurate value and the Prony's approximation (with 30 terms) of $\phi'(t)$. In Fig.2 are shown the reflection coefficient due to our ABC and a complete comparison with the Mur's first order ABC and Bi et al's DBC. Where v_0 in the Mur's ABC is sampled at the frequency f = 35 GHz, and v_1 and v_2 in Bi et al's DBC are picked out at the frequencies f_1 = 27.5 GHz and f_2 = 35 GHz. As observed, our ABC exhibits highly absorbing characteristics over the preset frequency band (25.0 ~ 45.0 GHz) although the waveguide is extremely dispersive.

The extension of our ABC into microstrips and coplanar waveguides is straightforward and is currently under way.

CONCLUSIONS

In conclusion, A novel wide-band absorbing boundary condition for FDTD method simulations has been proposed. Our ABC is based on the use of basic relationships between transverse fields of a guided travelling wave without reflection. It ensures that a wave is highly absorbed over the total frequency band of interest instead of for specific cases as done in conventional ABCs. The Prony's approximation to the phase factor in the time domain makes our ABC operator possess the recurrence property and yields an efficient implementation of our ABC in the FDTD simulations. The numerical experiments have shown that the present ABC has the superior absorbing performance over an extremely wide frequency band.

REFERENCES

[1] G. Mur, IEEE Trans Electromagn. Compat., vol.23, pp. 1073-1077, 1981
[2] B. Engquist et al, Math. Comput., vol.31, pp.629-651, 1977
[3] T.G. Moore et al, IEEE Trans Antennas and Propagat., vol.36, pp.1797-1812, 1988
[4] Z.Q. Bi et al, IEEE Trans MTT, vol.40, pp.774-777, 1992
[5] J-. P. Berenger, Jour. of Comput. Phys., vol.114, pp.185-200, 1994
[6] F.B. Hildebrand, *Introduction to Numerical Analysis*, New York, McGraw-Hill, 1956
[7] R.J. Luebbers et al, IEEE Trans Antennas and Propagat., vol.39, pp.29-34, 1991

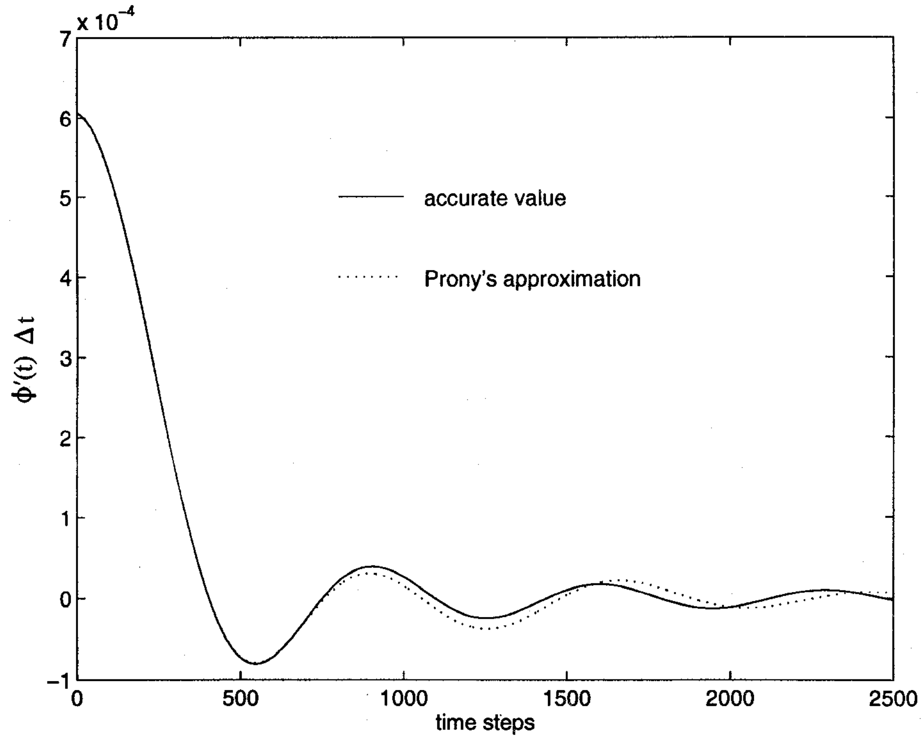

Fig.1 Accurate value and Prony's approximation of the phase factor $\phi'(t)$

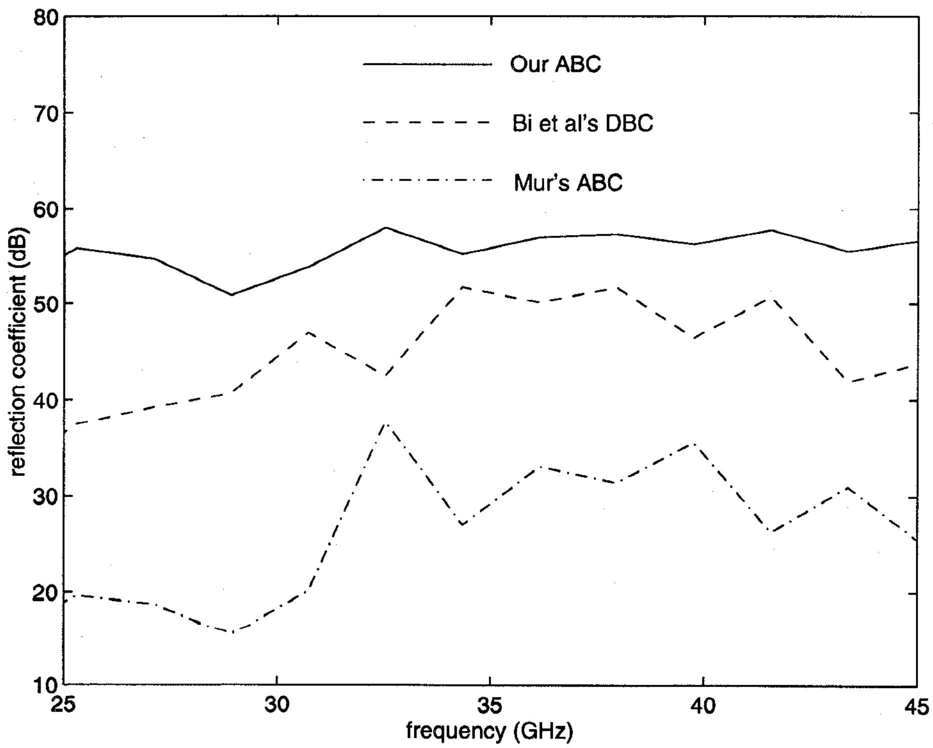

Fig.2 Reflection coefficient due to our ABC and complete comparison with Mur's first order ABC [1] and Bi et al's DBC [4]